بعض النشور المحدودة بجوار $x_0 = 0$

01°) $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} + o(x^{n+1}) = \sum_{k=0}^{n} \frac{x^k}{k!} + o(x^{n+1})$

02°) $shx = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots + \frac{x^{2n+1}}{(2n+1)!} + o(x^{2n+3}) = \sum_{k=0}^{n} \frac{x^{2k+1}}{(2k+1)!} + o(x^{2n+3})$

03°) $\operatorname{ch}x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots + \frac{x^{2n}}{(2n)!} + o(x^{2n+2}) = \sum_{k=0}^{n} \frac{x^{2k}}{(2k)!} + o(x^{2n+2})$

04°) $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + o(x^{2n+3}) = \sum_{k=0}^{n} \frac{(-1)^k}{(2k+1)!} x^{2k+1} + o(x^{2n+3})$

05°) $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots + (-1)^n \frac{x^{2n}}{(2n)!} + o(x^{2n+2}) = \sum_{k=0}^{n} \frac{(-1)^k}{(2k)!} x^{2k} + o(x^{2n+2})$

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06°) $(1+x)^\alpha = 1 + \alpha x + \frac{\alpha(\alpha-1)}{2!} x^2 + \cdots + \frac{\alpha(\alpha-1)\ldots(\alpha-n+1)}{n!} x^n + o(x^{n+1})$

$= 1 + \sum_{k=1}^{n} \frac{\alpha(\alpha-1)\ldots(\alpha-k+1)}{k!} x^k + o(x^{n+1}) \; ; \; \alpha \in \mathbb{R}$

07°) $\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots + x^n + o(x^{n+1}) = \sum_{k=0}^{n} x^k + o(x^{n+1})$

08°) $\frac{1}{x+1} = 1 - x + x^2 - x^3 + \cdots + (-1)^n x^n + o(x^{n+1}) = \sum_{k=0}^{n} (-1)^k x^k + o(x^{n+1})$

09°) $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots + (-1)^{n-1} \frac{x^n}{n} + o(x^{n+1}) = \sum_{k=1}^{n} \frac{(-1)^{k-1}}{k} x^k + o(x^{n+1})$

10°) $\sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \cdots + (-1)^{n-1} \frac{1\times3\times5\times\ldots\times(2n-3)}{2^n\, n!} x^n + o(x^{n+1})$

$= 1 + \sum_{k=1}^{n} (-1)^{k-1} \frac{1\times3\times5\times\ldots\times(2k-3)}{2^k\, k!} x^k + o(x^{n+1})$

11°) $\frac{1}{\sqrt{1+x}} = 1 - \frac{x}{2} + \frac{3}{8} x^2 - \cdots + (-1)^n \frac{1\times3\times5\times\ldots\times(2n-1)}{2^n\, n!} x^n + o(x^{n+1})$

$= 1 + \sum_{k=1}^{n} (-1)^k \frac{1\times3\times5\times\ldots\times(2k-1)}{2^k\, k!} x^k + o(x^{n+1})$

12°) $\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \cdots + (-1)^n \frac{x^{2n+1}}{2n+1} + o(x^{2n+3}) = \sum_{k=0}^{n} \frac{(-1)^k}{2k+1} x^{2k+1} + o(x^{2n+3})$

13°) $\operatorname{argth}x = x + \frac{x^3}{3} + \frac{x^5}{5} + \cdots + \frac{x^{2n+1}}{2n+1} + o(x^{2n+3}) = \sum_{k=0}^{n} \frac{x^{2k+1}}{2k+1} + o(x^{2n+3})$

14°) $\arcsin x = x + \frac{1}{2}\frac{x^3}{3} + \frac{3}{8}\frac{x^5}{5} + \cdots + \frac{1\times3\times5\times\ldots\times(2n-1)}{2^n\, n!} \frac{x^{2n+1}}{2n+1} + o(x^{2n+3})$

$= x + \sum_{k=1}^{n} \frac{1\times3\times5\times\ldots\times(2k-1)}{2^k\, k!} \frac{x^{2k+1}}{2k+1} + o(x^{2n+3})$

15°) $\operatorname{argsh}x = x - \frac{1}{2}\frac{x^3}{3} + \frac{3}{8}\frac{x^5}{5} - \cdots + (-1)^n \frac{1\times3\times5\times\ldots\times(2n-1)}{2^n\, n!} \frac{x^{2n+1}}{2n+1} + o(x^{2n+3})$

$= x + \sum_{k=1}^{n} (-1)^k \frac{1\times3\times5\times\ldots\times(2k-1)}{2^k\, k!} \frac{x^{2k+1}}{2k+1} + o(x^{2n+3})$

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16°) $\tan x = x + \frac{1}{3} x^3 + \frac{2}{15} x^5 + \frac{17}{315} x^7 + o(x^9)$

17°) $\operatorname{th}x = x - \frac{1}{3} x^3 + \frac{2}{15} x^5 - \frac{17}{315} x^7 + o(x^9)$